

D0-Branes As Confined Quarks ¹

Amir H. Fatollahi

Institute for Advanced Studies in Basic Sciences (IASBS),

P.O.Box 45195-159, Zanzan, IRAN

and

Institute for Studies in Theoretical Physics and Mathematics (IPM),

P.O.Box 19395-5531, Tehran, IRAN

fath@theory.ipm.ac.ir

Abstract

The possibility of using the quantum mechanics of D0-branes for the bound-states of quarks and QCD strings is investigated. Issues such as the inter D0-branes potential, the whiteness of the D0-branes bound-states and the large- N limit of D0-branes effective theory are studied. A possible role of the non-commutativity of relative distances of D0-branes in a study of ordinary QCD is discussed.

D0-branes are defined as particles which strings end on them [1, 2]. The question is: Can one use D0-branes dynamics as the effective theory of bound-states of quarks and QCD strings (QCD electric fluxes)? We study the following issues to approach this question:

- Inter D0-branes potential to compare with phenomenological one and that of electric-flux picture.
- Whiteness of D0-branes bound-states under $SU(N)$ electric field.
- Large- N behaviour of D0-branes bound-states to compare with QCD baryonic states at large- N .

Also a possible role of involving non-commutativity like the same one in relative distances of D0-branes in a study of ordinary QCD is discussed. Discussions here are mostly coming from the results appeared in [3, 4, 5].

¹Talk presented at *Isfahan String Workshop 2000, May 13-14, IRAN*.

Dynamics of N D0-branes is given by the matrix quantum mechanics resulted from dimensional reduction of $U(N)$ gauge theory to 0+1 dimension, by replacements $A_i \rightarrow X_i$ [6]: ²

$$S = \int dt m_0 \text{Tr} \left(\frac{1}{2} D_t X_i^2 + \frac{[X_i, X_j]^2}{4(2\pi\alpha')^2} \right), \quad (1)$$

$$i, j = 1, \dots, d, \quad D_t * = \partial_t * - i[a_0, *],$$

with $\frac{1}{2\pi\alpha'}$ =string tension ($l_s = \sqrt{\alpha'}$ and g_s =string coupling). X 's are in the algebra by the usual expansion $X_i = x_{i(a)} T_{(a)}$, $(a) = 1, \dots, N^2$. ³

The action is invariant under gauge transformations:

$$\begin{aligned} \vec{X} &\rightarrow \vec{X}' = U \vec{X} U^\dagger, \\ a_0 &\rightarrow a'_0 = U a_0 U^\dagger + i U \partial_t U^\dagger, \end{aligned} \quad (2)$$

with U as arbitrary unitary matrix, and consequently one finds:

$$\begin{aligned} D_t \vec{X} &\rightarrow D'_t \vec{X}' = U (D_t \vec{X}) U^\dagger, \\ D_t D_t \vec{X} &\rightarrow D'_t D'_t \vec{X}' = U (D_t D_t \vec{X}) U^\dagger. \end{aligned} \quad (3)$$

D0-branes are presented “classically” by diagonal matrices and the action takes the form of N free particles for them:

$$S = \int dt \sum_{(a)=1}^N \frac{1}{2} m_0 \dot{x}_{(a)}^2. \quad (4)$$

The action is non-relativistic, but can be used for covariant formulation by Light-Cone Frame (LCF) interpretation with the following identifications [7, 8]:

$$m_0 = p^+, \quad t = x^+, \quad X_i = \text{transverse directions}. \quad (5)$$

By the scalings [9]

$$t \rightarrow g_s^{-1/3} t, \quad a_0 \rightarrow g_s^{1/3} a_0, \quad X \rightarrow g_s^{1/3} X, \quad (6)$$

one finds the relevant energy and size scales as:

$$E \sim g_s^{1/3} / l_s, \quad l_{d+2} = g_s^{1/3} l_s. \quad (7)$$

²Here we ignore supersymmetry. Also we work in arbitrary dimensions d .

³To avoid confusion, we put group indices always in ().

The length l_{d+2} should be identified as the fundamental length scale of the covariant $d+2$ dimensional theory which is expected that its LCF formulation is presented by the action (1). So we take (for $d=2$) l_{d+2} as the inverse of 4 dimensional QCD mass scale, denoted by Λ_{QCD} .

1. D0-Branes Potential: The effective potential between D0-branes comes from the effect of quantum fluctuations around a classical configuration, presented here by diagonal matrices. This work is equivalent with integrating over oscillations of strings stretched between D0-branes. One-loop effective action is given by $(2\pi\alpha' = 1)$ [10]:

$$\left(\int dt\right)V(X_\mu^{cl}) = \frac{1}{2}\text{Tr}\log\left(P_\lambda^2\delta_{\mu\nu} - 2iF_{\mu\nu}\right) - \text{Tr}\log\left(P_\lambda^2\right), \quad (8)$$

with

$$\begin{aligned} P_\mu^* &\equiv [X_\mu^{cl}, *], & F_{\mu\nu}^* &\equiv [f_{\mu\nu}, *], & f_{\mu\nu} &\equiv [X_\mu^{cl}, X_\nu^{cl}], \\ \mu, \nu &= 0, 1, \dots, d, & X_0 &= i\partial_t + a_0, \\ P_\lambda^2 &= -\partial_t^2 + \sum_{i=1}^d P_i^2, & \text{for } a_0^{cl} &= 0. \end{aligned} \quad (9)$$

For two static D0-branes at distance r we may take:

$$\begin{aligned} X_1^{cl} &= \frac{1}{2} \begin{pmatrix} r & 0 \\ 0 & -r \end{pmatrix}, & X_0^{cl} &= i\partial_t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\ a_0^{cl} &= X_{i>1}^{cl} = 0. \end{aligned} \quad (10)$$

So one finds

$$P_1 = \frac{r}{2} \otimes \Sigma_3, \quad P_0 = i\partial_t \otimes 1_4, \quad P_{i>1} = 0, \quad (11)$$

with $\Sigma_3^* = [\sigma_3, *]$ which has 0, 0, ± 2 as eigenvalues. Also we find the operator $P_\lambda^2 = -\partial_t^2 \otimes 1_4 + \frac{r^2}{4} \otimes \Sigma_3^2$ as a harmonic oscillator operator with frequency $\omega \sim r/\alpha'$. One-loop is a good approximation for $\omega \gg m_0\dot{r}^2$ or $rg_s \gg l_s\dot{r}^2$ which for $g_s \rightarrow 0$ ($m_0 \gg l_s^{-1}$) is satisfied for large separations and low velocities.

One-loop effective action can be calculated easily to find:

$$\begin{aligned} V(r) &= \left(\frac{d-1}{2}\right)\text{Tr}\log\left(P_\lambda^2\right) \\ &= -2\left(\frac{d-1}{2}\right)\int_0^\infty \frac{ds}{s} \int_{-\infty}^\infty dk_0 e^{-s(k_0^2+r^2)} \\ &\quad + \text{traces independent of } r, \end{aligned} \quad (12)$$

and after integrations one obtains

$$\begin{aligned}
V(r) &= -2\left(\frac{d-1}{2}\right) \int_0^\infty \frac{ds}{s} \left(\frac{\pi}{s}\right)^{\frac{1}{2}} e^{-sr^2} \\
&= 4\pi\left(\frac{d-1}{2}\right) |r| \\
&\quad -\infty \text{ (independent of } r\text{)}.
\end{aligned} \tag{13}$$

which is the linear potential with phenomenology interests [11, 12]. So the effective theory for the relative dynamics of two D0-branes is given by:

$$S = \int dt \left(\frac{1}{2} \frac{m_0}{2} \dot{\vec{r}}^2 - 4\pi \left(\frac{d-1}{2} \right) \frac{|\vec{r}|}{2\pi\alpha'} \right) \tag{14}$$

and one finds the energy scale as $E \sim \alpha'^{-2/3} m_0^{-1/3} \sim g_s^{1/3}/l_s$, as pointed in eq. (7). By assuming the dynamics in LCF with the longitudinal momentum m_0 , we have $M^2 \sim p^+ p^- \sim m_0 E \sim g_s^{-2/3} l_s^{-2} \sim l_{d+2}^{-2}$, by eq. (7).

This potential is also true for every pair of D0-branes, and one can write the effective theory for N D0-branes as:

$$S = \int dt \left(\frac{1}{2} m_0 \sum_{(a)=1}^N \dot{\vec{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \sum_{(a)>(b)=1}^N \frac{|\vec{r}_{(a)} - \vec{r}_{(b)}|}{2\pi\alpha'} \right). \tag{15}$$

In a recent work [13] by taking the linear potential in transverse directions of LCF between the quarks of a baryonic state, the structure functions are obtained with a good agreement with observed ones.

One can relate the parameter $1/\alpha'$ in the front of the potential to gauge theory parameters. To do so one needs a string theoretic description of the gauge theory in LCF, and the natural guess for this is “Light-Cone–lattice gauge theory” (LClgt) [14]. In LClgt one assumes time direction and one of the spatial directions to be continuous to define LC variables $x^\pm \sim t \pm z$. Other spatial directions play the role of transverse directions of LCF which are assumed to be lattices. As usual in LCF, time is x^+ and continuous and so we have a Hamiltonian formulation [15] of lgt [16]. The linear potential in LClgt, related to string tension is known to be:

$$V(r) \sim \frac{g_{YM}^2}{a^2} |\vec{r}|, \tag{16}$$

with a as the lattice spacing parameter in the transverse directions. Via this one finds the relation

$$\frac{1}{\alpha'} \sim \frac{g_{YM}^2}{a^2}, \tag{17}$$

for the parameters.

2. Whiteness: To find the charge and colour of D0-branes bound-states we need to know their dynamics in YM backgrounds. In the case of electromagnetism there is a simple relation:

$$m_0 \ddot{\vec{x}} = q(\vec{E}_{ext.} + \vec{v} \times \vec{B}_{ext.}). \quad (18)$$

The concept of gauge invariance here is understood as the invariance of the equations of motion under the gauge symmetry transformations. In the case of chromodynamics in r.h.s. matrices in adjoint representation are placed and so they transform like:

$$\vec{E} \rightarrow \vec{E}' = U \vec{E} U^\dagger, \quad \vec{B} \rightarrow \vec{B}' = U \vec{B} U^\dagger. \quad (19)$$

So we need to replace the l.h.s. with matrices with the correct behaviour under gauge transformations. Now we have good candidate for non-commutative coordinates: D0-branes coordinates. One may write for “matrix” coordinates

$$m_0 \ddot{\vec{X}} = q(\vec{E}_{ext.} + \dot{\vec{X}} \times \vec{B}_{ext.}), \quad (20)$$

but yet the l.h.s. does not have correct behaviour under gauge transformations! Here the world-line gauge symmetry eq. (2) helps us, to write the generalized “Lorentz” equation as

$$m_0 D_t D_t \vec{X} = q(\vec{E}_{ext.} + D_t \vec{X} \times \vec{B}_{ext.}), \quad (21)$$

and now by eq. (3) both sides have an equal behaviour under gauge transformations. The space dependence of the fields is a subtle point, because the coordinates themselves change under transformation on the gauge fields $A_{(a)}$ ’s. Resolving the space dependence may be done by assuming the D0-branes bound-states very small and then taking the space dependence of external fields just for the centre-of mass (c.m.) (see fig. below). The coordinates and momenta of c.m. are given by the trace of matrices, as:

$$\vec{x}_{cm} \equiv \frac{1}{N} \text{Tr} \vec{X}, \quad \vec{p}_{cm} \equiv \text{Tr} \vec{P}, \quad (22)$$

and so are invariant under transformations $X \rightarrow UXU^\dagger$. So by taking the space dependence just for c.m. we have:

$$\vec{E}_{ext} = \vec{E}_{ext}(x_{cm}), \quad \vec{B}_{ext} = \vec{B}_{ext}(x_{cm}). \quad (23)$$

To specify the charge or colour of an extended object (e.g. a bound-state), we study the dynamics in absence of magnetic field ($\vec{B} = 0$) and in uniform electric field

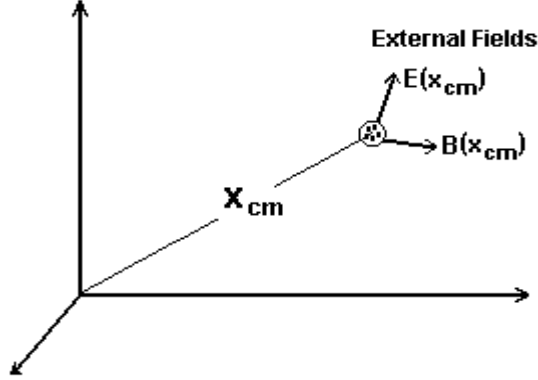


Figure 1: *Space dependence of the external fields.*

($\vec{E}(x) = \vec{E}_0$). In our case the c.m. dynamics decouples from non-Abelian parts due to the trace nature of $U(1)$ and $SU(N)$ parts. So we have:

$$m_0 \ddot{\vec{x}}_{c.m.} = q \vec{E}_{(1)0ext.}, \quad (24)$$

which (1) is for $U(1)$ part of $U(N)$. So the dynamics of c.m. will not be affected by the non-Abelian part: the c.m. is white. It means that each D0-brane sees the net effect of other D0-branes as the white-complement of its colour: the field fluxes extracted from one D0-brane to other ones are as the same of one flux between a colour and an anti-colour. The linear potential of previous part is consistent with flux-string picture. The number of D0-branes in the bound-state is equal to the same of baryons, N .

3. Large- N : Baryons show special properties at large- N limit of gauge theories [17]:

- Their mass grows linearly by N .
- Their size is not dependent on N . So their density goes to infinity at large- N .
- Baryon-baryon force grows in proportion to N .

These properties are mainly extracted from the study of quantum mechanics of N quarks and their bound-states as an N -body problem. The problem is approached by approximations (e.g. Hartree) for general potentials which have two characters: 1) they are attractive, and 2) their strength decreases by $1/N$ at large- N . Here we check the same behaviours for our problem, by reminding LCF interpretations. The

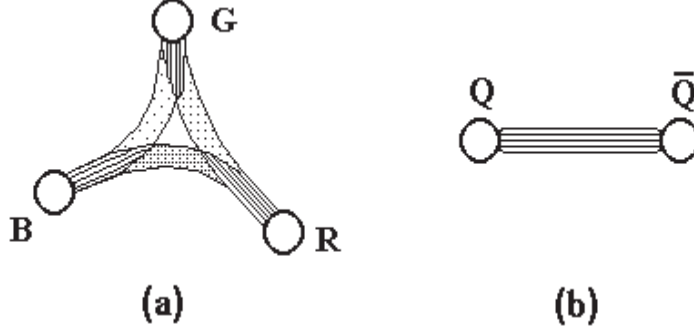


Figure 2: *The net electric flux extracted from each quark is equivalent in a baryon (a) and a meson (b). The D0-brane-quark correspondence suggests the string-like shape for fluxes inside a baryon (a).*

effective theory for N D0-branes is obtained to be

$$S = \int dt \left(\frac{1}{2} m_0 \sum_{(a)=1}^N \dot{\vec{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \sum_{(a)>(b)=1}^N \frac{|\vec{r}_{(a)} - \vec{r}_{(b)}|}{2\pi\alpha'} \right). \quad (25)$$

with the relation $1/\alpha' \sim g_{YM}^2/a^2$. Also we have the replacement at large- N :

$$g_{YM} \xrightarrow{\text{large } N} \frac{g_{YM}}{\sqrt{N}}, \quad (26)$$

and so the action is read

$$S = \int dt \left(\frac{1}{2} m_0 \sum_{(a)=1}^N \dot{\vec{r}}_{(a)}^2 - 4\pi \left(\frac{d-1}{2} \right) \frac{g_{YM}^2}{a^2} \frac{1}{N} \sum_{(a),(b)=1}^N |\vec{r}_{(a)} - \vec{r}_{(b)}| \right). \quad (27)$$

The associated Hamiltonian of this action is the same used before [17] except for the potential term, which is Coulomb one there. Here we just check the mass: The kinetic term of c.m. ($\frac{\vec{P}^2}{Nm_0}$) grows with N , and the net potential for each D0-brane takes a factor $\frac{1}{2}N(N-1)$ due to pair interactions. So the potential term grows with $\frac{1}{2}N(N-1)g_{YM}^2/N \sim N$. The energy grows as $E \sim N$ at large- N . In LCF the energy is P^- . Also the total longitudinal momentum of this bound-state is $P^+ = Np^+$ with $p^+ = m_0$. So the invariant mass M is read

$$M^2 = 2P^+P^- - \vec{P}^2 \sim N^2 \Rightarrow M \sim N. \quad (28)$$

Space-Time Considerations: Non-Commutativity

Relative coordinates of D0-branes are matrices and so non-commutative. If the correspondence between the dynamics of D0-branes and confined quarks has a root in Nature, the question will be about possible justification of this non-commutativity. In the following 3 comments are in order:

1. *Special Relativity Idea:* In the way to find a consistent theory for the propagation of electromagnetic fields, special relativity learns to us that space and time should be treated as a 4-vector X_μ under boost transformations, such as the gauge field 4-vector, A_μ .

Also the idea of supersymmetry (SUSY) can be considered as a natural continuation of the special relativity program: Adding spin half sector to the coordinates of space-time as the representative of the fermions of the Nature. This idea leads one to the super-space formulation of SUSY theories. Also it is the same way which one introduces fermions to the bosonic string theory.

Now, what may be modified if in some regions of space and time there exists non-Abelian (non-commutative) gauge fields? In the present Nature non-Abelian gauge fields can not make spatially long coherent states; they are confined or too heavy. But the picture may be changed inside a hadron. In fact recent developments of string theories sound this change and it is understood that non-commutative coordinates and non-Abelian gauge fields are two sides of one coin. As we discussed, the interaction between D-branes is the result of path-integrations over fluctuations of the non-commutative parts of coordinates. It means that in this picture “non-commutative” fluctuations of space-time are the source of “non-Abelian” interactions. One may summarize this discussion as in the table below:

Field \leftrightarrow Space – Time		
A_μ (Photons)	\leftrightarrow X_μ (4 – Vector)	: Maxwell
ψ (Fermions)	\leftrightarrow $\theta, \bar{\theta}$ (Super Coordinates)	: SUSY
$A_\mu^{(a)}$ (Gluons)	\leftrightarrow $X_\mu^{(a)}$ (Matrix Coordinates)	: QCD

As it has been mentioned previously, the non-commutativity of D0-branes coordinates just come back to their relative distances and the c.m.’s of different bound-states of D0-branes presented by the trace of the position matrices, are commutative objects. We know that QCD fields are zero outside of hadrons, so the non-commutativity should be restricted to relative distances of hadron constituents (see fig.).

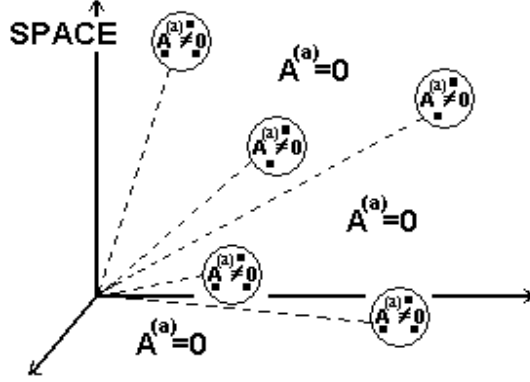


Figure 3: *Non-commutativity inside the hadrons is just because of the different nature of fields in them. The c.m.'s are presented by dotted lines.*

2. *Recent Example:* Pure U(1) gauge theory on ordinary space has free photons. On non-commutative space the theory has interacting photons and the structure of the theory becomes very similar to the same of non-Abelian gauge theory, summarized in the table below [18]:

$$\begin{aligned}
\text{Commutative Space} &\rightarrow \text{Non - Commutative Space} \\
[X, X] = 0 &\rightarrow [X, X] = i\theta \\
\text{free photons} &\rightarrow \text{interacting photons} \\
F = \partial A &\rightarrow F = \partial A + g\{A, A\} \\
F' = F &\rightarrow F' = U * F * U^{-1}.
\end{aligned} \tag{29}$$

The lesson of this example is that one may cover the aspects of non-Abelian gauge theories by changing the structure of space-time. It means that by assuming non-commutativity between the coordinates of space-time one can get a theory with properties similar to non-Abelian theories.

For our special case the question will be about “Is the structure of space-time suggested by D0-brane–quark correspondence appropriate to cover the non-Abelian structure of $U(N)$ gauge theories or QCD?”

3. *Lattice Continuum Limit:* Firstly let us have a look to the procedure of taking the continuum limit of lattice gauge theories ⁴. Consider the correlation length ξ

⁴This discussion is borrowed from [19].

between two plaquettes:

$$\xi = a \xi_{latt} \quad (30)$$

which is expressed in terms of dimensionless parameter ξ_{latt} , and lattice spacing parameter a , appearing just as a scale factor. ξ is the physical quantity and in the continuum limit it should remain constant, providing:

$$\xi_{latt} \xrightarrow{a \rightarrow 0} \infty. \quad (31)$$

The correlation function has the behaviour:

$$G(r) \xrightarrow{r \rightarrow \infty} e^{-r/\xi}, \quad (32)$$

and by setting $r = na$, the ξ_{latt} does not depends on a , becoming a function of coupling constant g :

$$\xi_{latt} = - \lim_{n \rightarrow \infty} \frac{n}{\ln G(n)} = f(g), \quad (33)$$

which we have for it at a critical value g_c :

$$f(g_c) = \infty. \quad (34)$$

One can find the g -dependence of any physical quantity by the function $f(g)$. Assume Q is a physical quantity with dimension $[\text{length}]^d$; putting $Q = a^d Q_{latt}$ we have:

$$Q \xi^{-d} = Q_{latt} \xi_{latt}^{-d}. \quad (35)$$

Assuming $Q \xi^{-d}$ is finite in the continuum limit we have:

$$Q_{latt} = C [f(g)]^d, \quad (36)$$

with C as a constant.

One can find the behaviour of the function $f(g)$ at the strong and weak coupling limit. At strong coupling limit lattice gauge theory gives the string tension K with dimension $[\text{length}]^{-2}$, so:

$$f(g) = \frac{1}{(a^2 K)^2} = \frac{C}{\ln^2(\kappa g)}, \quad g \gg 1, \quad (37)$$

At weak coupling we have the perturbative result as:

$$\frac{g^2}{4\pi} = \frac{1}{\gamma_0 \ln(M^2/\Lambda^2)}, \quad \gamma_0 = \frac{33}{12\pi}, \quad (38)$$

where M is a mass scale. So we have:

$$f(g) = C' e^{[1/(2\gamma_0 g^2)]} = C' e^{[8\pi^2/(11g^2)]}, \quad g \ll 1. \quad (39)$$

These two behaviours are plotted in the figure, and one can see that the continuum limit ($f(g) = \infty$) is just gained at $g_c = 0$. Based on this, for every finite value of the coupling constant lattice formulation does not reach to continuum limit.

On the other hand, we know that the natural framework of formulation theories on discrete space-time is Non-Commutative Geometry, with the known examples two-point world or the standard model of particles on two-sheet world [20][21].

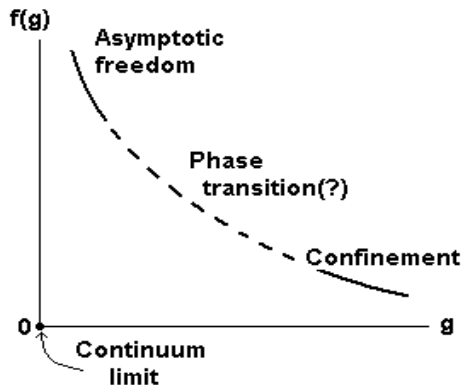


Figure 4: Plot of $f(g)$ versus coupling g . The continuum limit is just gained at exactly zero coupling.

References

- [1] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724, hep-th/9510017.
- [2] J. Polchinski, “*Tasi Lectures On D-Branes*”, hep-th/9611050.
- [3] A.H. Fatollahi, “*Do Quarks Obey D-Brane Dynamics?*”, hep-ph/9902414.
- [4] A.H. Fatollahi, “*Do Quarks Obey D-Brane Dynamics? II*”, hep-ph/9905484.

- [5] A.H. Fatollahi, “*D0-Branes As Light-Front Confined Quarks*”, hep-th/0002021.
- [6] E. Witten, Nucl. Phys. **B460** (1996) 335, hep-th/9510135.
- [7] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043.
- [8] L. Susskind, “*Another Conjecture About M(atrix) Theory*”, hep-th/9704080.
- [9] D. Kabat and P. Pouliot, Phys. Rev. Lett. **77** (1996) 1004, hep-th/9603127; U.H. Danielsson, G. Ferretti and B. Sundborg, Int. J. Mod. Phys. **A11** (1996) 5463, hep-th/9603081.
- [10] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. **B498** (1997) 467, hep-th/9612115.
- [11] W. Lucha, F. Schoberl and D. Gromes, Phys. Rep. **200** (1991) 127; S. Mukherjee, et. al., Phys. Rep. **231** (1993) 201.
- [12] W. Kwong, J.L. Rosner and C. Quigg, Ann. Rev. Nucl. Part. Sci. **37** (1987) 325.
- [13] G.S. Krishnaswami, “*A Model Of Interacting Partons For Hadronic Structure Functions*”, hep-ph/9911538.
- [14] W.A. Bardeen and R.B. Pearson, Phys. Rev. **D14** (1976) 547; W.A. Bardeen, R.B. Pearson and E. Rabinovici, ibid **21** (1980) 1037; S. Dalley and B. van de Sande, Phys. Rev. **D59** (1999) 065008, hep-th/9806231, and references therein.
- [15] J.B. Kogut and L. Susskind, Phys. Rev. **D11** (1975) 395; J.B. Kogut, Rev. Mod. Phys. **55** (1983) 775.
- [16] K.G. Wilson, Phys. Rev. **D10** (1974) 2445.
- [17] E. Witten, Nucl. Phys. **B160** (1979) 57.
- [18] A. Connes, M.R. Douglas and A. Schwarz, JHEP **9802** (1998) 003, hep-th/9711162.
- [19] K. Huang, “*Quarks, Leptons And Gauge Fields*”, World Scientific, 2nd edition (1992) pp. 303-305.
- [20] A. Connes, “*Noncommutative Geometry*”, Academic Press (1994).
- [21] A. Dimakis, F. Muller-Hoissen and T. Striker, J. Phys. **A26** (1993) 1927.